Validity of classical scaling laws in laminar channel flow with periodic spacer-like obstacles

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Introduction and motivation

Laminar channel flows with periodic obstacles are present in many heat and mass transfer applications, e.g. membrane technologies such as electro-dialysis or spirally wound membrane modules (see Fig. 1). For process design, classical scaling laws are typically used, which scale the transfer (Sherwood) number, *Sh*, to the hydrodynamic Reynolds number, *Re*, the fluid specific Schmidt number, *Sc*, and to some dimensionless geometric parameters, D_h, in a classical form like

$$Sh = C \cdot Re^{\alpha} \cdot Sc^{\beta} \cdot D_h^{\gamma}.$$

However, the validity of those scaling laws is limited to regions where the concentration boundary layer develops. It is well known that the transfer numbers approach a constant (Reynolds and Schmidt independent) value in the developed region of a laminar channel flow.

This study examines numerically the validity of the scaling laws if the channel flow is interrupted periodically by cylindrical obstacles of different size and separation distance.

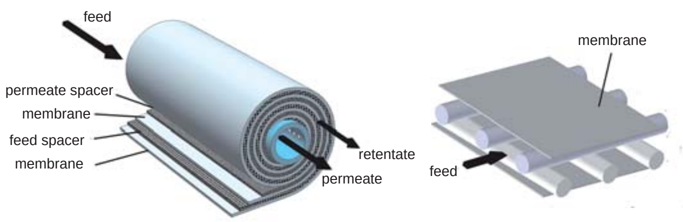

Fig. 1: Schematic of spiral-wound membrane module showing the permeate and feed spacers between the membranes (left) and a close-up of a spacer-filled membrane channel (figures adopted from Beale et al., *JHT*, 2013).

Transfer in empty channel flows: Graetz problem

Transport processes in laminar channel flows with various cross-sectional geometries have been examined theoretically and analytically for many decades and is as such a classical problem in heat transfer books. The problem is segregated into the fully developed and developing flow regions (hydrodynamically and/or thermally).

Scaling laws in the developing region

For high Peclet numbers, if conduction in streamwise direction is negligible, the scalar transport equation reduces to a balance between axial convection and transverse conduction.

Within the thermal entrance length of a laminar flow, the thermal development is self-similar with respect to the so-called Graetz number:

$$1/Gz = x/(D_h \cdot Re \cdot Sc).$$

The self-similar behavior is presented in Fig. 2, revealing a decrease in the local transport number by the power of 1/3.

Local transport number correlations in the thermally developing region of the parallel-plate channel flow are given in Shah and London (1978):

$Nu_x = 1.490\, Gz^{1/3}$ $1/Gz < 0.0002$

$Nu_x = 1.490\, Gz^{1/3} - 0.4$ $0.0002 < 1/Gz < 0.001$

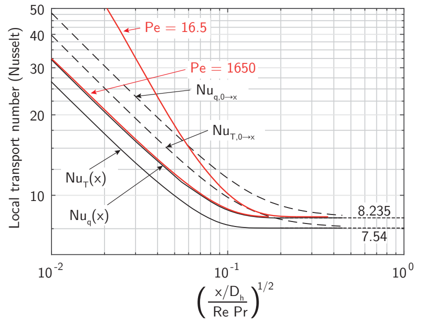

Fig. 2: Heat transfer in a parallel-plate duct: Black lines: Results for an initially fully developed velocity profile and a large Peclet number based on data presented in the book of Shah and London (1978). Red lines: Own results (DNS) for a Peclet number of Pe = 16.5 and Pe = 1650.

Self-similar behavior in spacer-filled channel flow

Despite the low flow rates in membrane channel flows, entrance length effects are generally believed to be "*less important when spacers are involved as they introduce a break-up of the boundary layer on the spacer dimension ...*" (Howell, 1993).

Figure 4 disproves this general statement by the results of fully resolved numerical simulations. Local and overall Nusselt numbers decay with the inverse Graetz number by the same power law as in the empty channel.

The root cause for the sustained scaling law is the flow structure shown in Fig. 3b). The flow is two-dimensional, stationary, and non-mixing. Streamlines are deformed by the cylinder and the fixed walls such that they approach each other, showing a local flow acceleration up to the position of maximal channel blockage ($x/D_h = 0$).

Figures 3 d) and f) illustrate the local temperature by isolines (top solid lines) and heatlines (bottom dashed lines) according to Bejan (2013). Heatlines have the advantage of illustrating the actual direction of energy flow driven by convection and conduction, whereas isotherms are only appropriate to illustrate the driving potential for conduction.

Heatlines and streamlines are quasi-identical, revealing that convective transport in flow direction is orders of magnitude larger than diffusive transport normal to the streamlines. Consequently, the temperature varies insignificantly in the streamwise direction, allowing for a separation of scales and a boundary layer-like approximation of the respective problem.

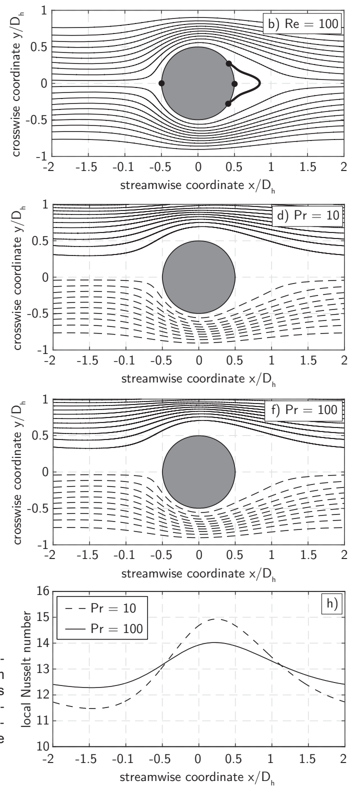

Fig. 3: Streamlines in the hydrodynamically and thermally fully developed region for Re = 100. Thick black lines indicate a stream function value of zero and encloses the separation zone. Black dots mark stagnation points. Isolines and heatlines are shown for the constant heat flux boundary condition. The lowest plot compares the local Nusselt number profile for the two different Prandtl numbers. The average Nusselt number of the two cases is: Nu(Pr = 10) = 12.56, Nu(Pr = 100) = 12.74.

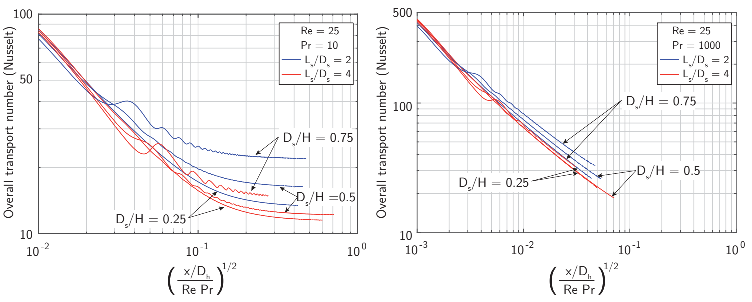

Fig. 4: Overall dimensionless transport coefficient in the entrance region of a parallel-plate duct with periodic obstacles for Re = 25 and a variation of the geometric expansion D_s/H = [0.25; 0.5; 0.75] and $L_s = D_s$ = [2; 4]. The constant heat flux boundary condition is applied.

Geometry, boundary conditions & dimensionless numbers

In dimensionless form, the problem is described by a set of parameters involving fluid properties, geometric ratios and the characteristic velocity:

Reynolds number:	$Re = u_0 D_h/\nu$
Prandtl number:	$Pr = \nu/\alpha$
Geometric ratios:	H/D_S and L_s/D_S
Nusselt number:	$h(x)D_h/k$

u_0:	mean velocity
D_h:	hydraulic diameter (D_h=2H)
ν:	kinematic viscosity
α:	thermal diffusivity
k:	thermal conductivity
H:	channel height
D_S:	spacer diameter
L_S:	spacer separation distance
h(x):	local heat transfer coefficient

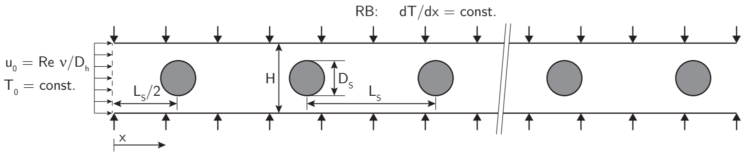

Fig. 5: Schematic drawing of a heated duct flow interrupted by periodically spaced obstacles. Constant flux boundary condition is applied on both sides of the channel.

Loss of self-similarity

In turbulent pipe flows, the entrance length is known to be independent from the Graetz number. Transition to turbulence is thus a clear limitation of the scaling law. However, the flow past a cylinder in unconfined and confined configurations undergoes a cascade of transition and allows to study the limit of self-similarity. One important transition is the onset of vortex shedding, leading to an instationary flow regime.

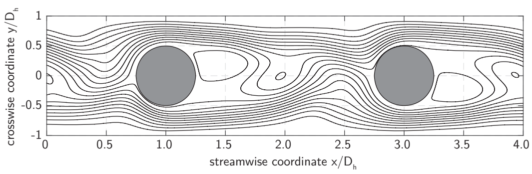

Fig. 6: Streamlines of the instantaneous flow field for Re = 400 with periodic obstacles (D_s = H = 0.5 and L_s = D_s = 4). Periodic, alternating vortices shed behind consecutive cylinders and wall attached vortices arise, which periodically shed from the wall and transport liquid convectively to the duct center.

The dimensionless transport number presented in Fig. 7 reveals a significantly different behavior compared to Fig. 4. The differences are:

- Self-similarity of the local and overall transport number with respect to the Prandtl number is lost.
- Significant entrance length effects are only present in the high Prandtl number case. For lower Prandtl numbers, the transport number decreases slightly over the channel length.
- The transport number in the fully developed region converges to different values depending on Prandtl number.

The entrance length behavior changes between the steady and unsteady regime.

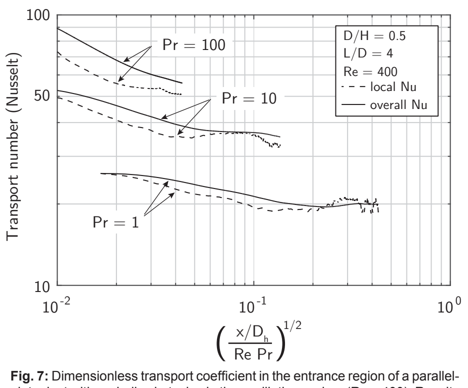

Fig. 7: Dimensionless transport coefficient in the entrance region of a parallel-plate duct with periodic obstacles in the oscillating regime (Re = 400). Results are obtained from an instantaneous temperature field.

Conclusions and outlook

- Graetz number scaling remains valid in stationary channel flows despite periodic obstacles
- Entrance length effects dominate transfer in spacer-filled channel flows for high Schmidt number
- Loss of self-similar behavior is associated with the transition to an unsteady flow regime
- Scaling laws for spacer filled channels need to be flow regime dependent
- A predictive model for the transition to the unsteady flow regime is required

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